CoSIR: Managing an Epidemic via Optimal Adaptive Control of Transmission Rate Policy

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Epidemic Control via Transmission Restrictions

Shaping an epidemic with an **adaptive contact restriction policy** is a critical challenge for public health officials that requires exploration.

- **Scenario-based forecasting** methods are not well-suited for control because these only permit limited exploration of scenarios.
- **Periodic lockdowns and RL techniques** do not sufficiently exploit the mathematical structure of epidemic dynamics.
- **Current economic epidemiological control formulations** focus on impact modeling, but are not easily tractable.

Problem Statement: Given total population (N), current susceptible population (S_{curr}), current infectious population (I_{curr}), a set of restriction levels ($A = \{a_i\}$) and a time horizon (T), identify a restriction schedule $[a_t], [t]_{curr+1}^{curr+T}$, s.t. infectious levels average I_{avg}^{target} and do not exceed I_{max}^{target}.

Contributions

- Novel mapping between SIR dynamics and Lotka-Volterra (LV) system under a specific transmission rate policy (LVSIR).
- Derivation of optimal control policy for transmission rate (CoSIR) using control-Lyapunov functions (CLF) based on the "Lotka-Volterra energy".
- Practical control algorithm that combines the CoSIR policy with statistical estimation of other model parameters & approximation to discrete levels.
- Evaluation on COVID-19 data to demonstrate efficacy and adaptability.

Optimal Control of SIR via Mapping to LV System

Optimal control of epidemic, i.e., regulating infection levels in SIR system has a direct analogy with population control in LV systems.

- Infectious population (I) $\leftrightarrow$ Predators (q): Inflow and outflow into infectious compartment are akin to birth and death of predators.
- Susceptible contacts (βS) $\leftrightarrow$ Prey (p): Susceptible contacts act as "nourishment" to infectious population.
- Exact equivalence requires a specific transmission rate β policy (LVSIR).

Control of non-linear dynamical systems often relies on control-Lyapunov functions. CoSIR follows a similar approach using "Lotka-Volterra energy".

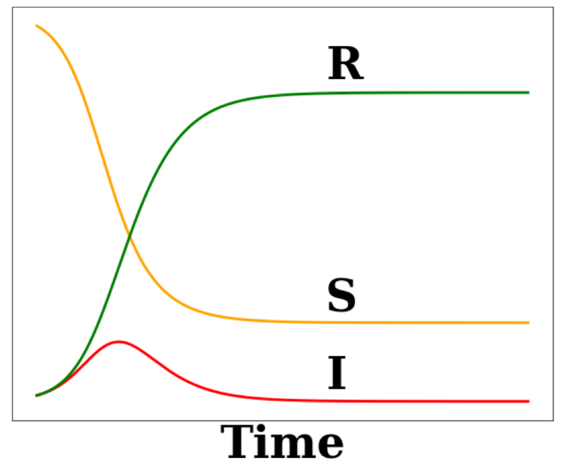
Interpretation of CoSIR β-control policy:

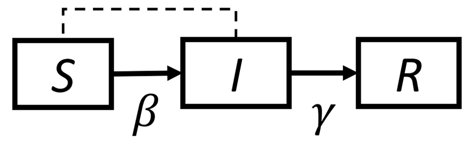
- $\beta^2 I/N$: Relaxation due to the decreasing susceptibility
- $(r - eI)\beta$: Stabilization but oscillatory behavior
- $u\beta$: Dissipation of energy and convergence to the equilibrium.

Properties of LVSIR System

- Stable equilibrium at $(J^*, I^*) = (\gamma N, r/e)$.
- Initialization at equilibrium $\Rightarrow$ constant (J, I) and linear S, R.
- Initial state different from equilibrium $\Rightarrow$ cyclic behaviour.
  - LV system "energy" $w(J, I) = \gamma(x - \log(x) - 1) + r(y - \log(y) - 1)$ remains constant where $x = J/J^*, y = I/I^*$.
  - I and J curves exhibit periodic oscillations resulting in a closed phase plot with extrema $\{(x_{min}, 1), (1, y_{min}), (x_{max}, 1), (1, y_{max})\}$where (x_{min}, x_{max}) and (y_{min}, y_{max}) satisfy $x - \log(x) = 1 + w_0\gamma$ and $y - \log(y) = 1 + \frac{w_0}{r}$ respectively.
  - In each cyclic period, S reduces by a fixed amount $\Delta S = \gamma I T_{period}$.

SIR

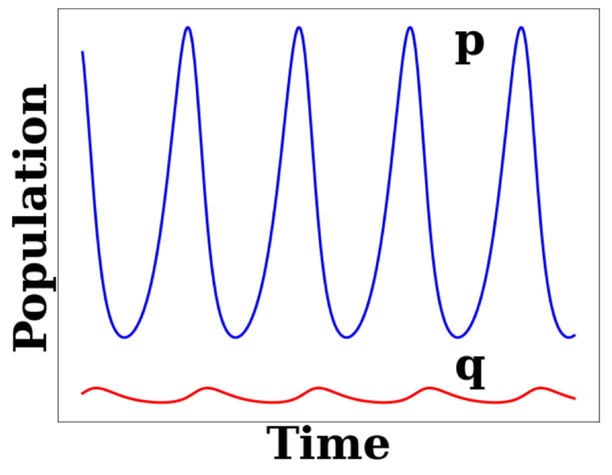


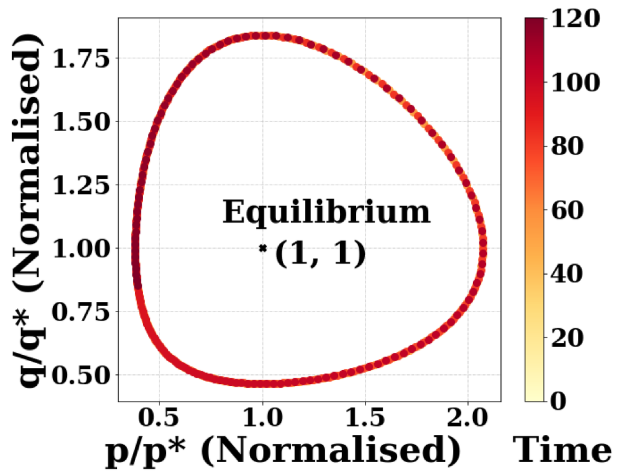


$$\dot{S} = \frac{-\beta IS}{N} \quad \dot{I} = \frac{\beta IS}{N} - \gamma I$$
$$\dot{R} = \gamma I \quad S + I + R = N$$

S: Susceptible
I: Infectious
R: Removed
N: Population
β: Transmission rate
γ: Inverse infectious period

LV

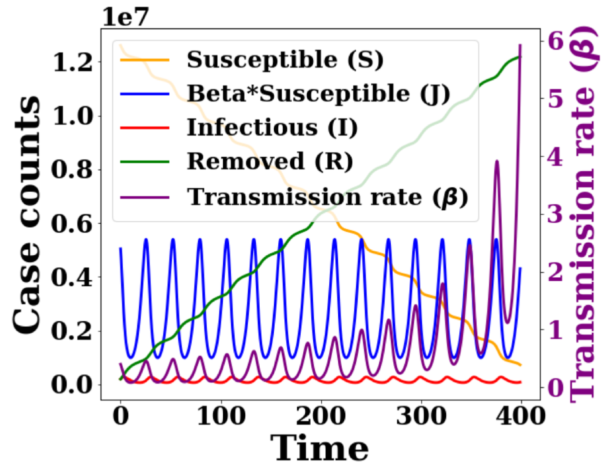


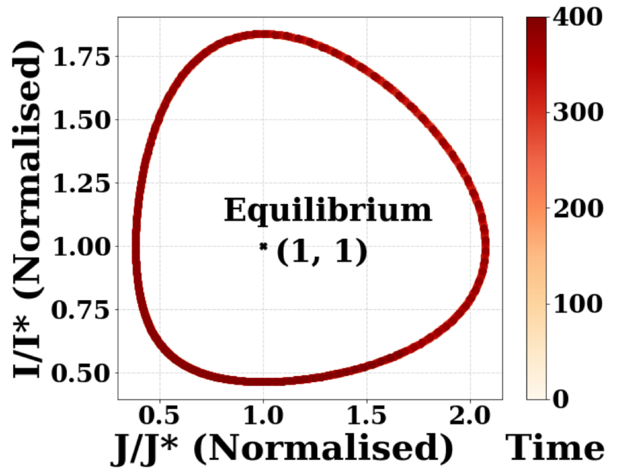


p: prey q: predator
r: growth rate of prey
e: prey consumption rate
b: predator birth rate
d: predator death rate

$$\dot{p} = rp - epq; \quad \dot{q} = bpq - dq$$
$$(p^*, q^*) = \left(\frac{d}{b}, \frac{r}{e}\right)$$

LVSIR

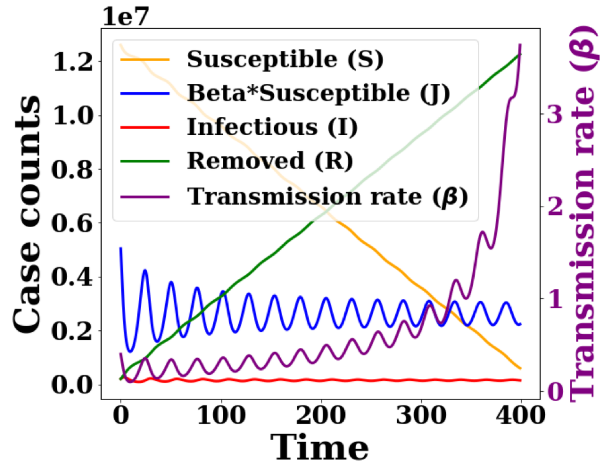


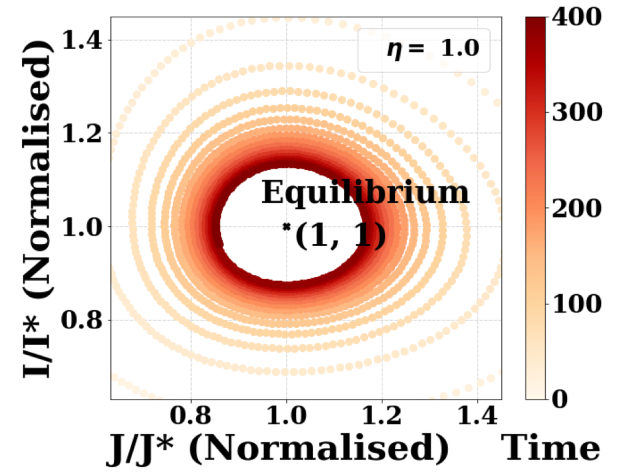


$J = \beta S$ (susceptible contacts)

$$J \to p, I \to q, b \to \frac{1}{N}, d \to \gamma$$
$$\dot{\beta} = \frac{\beta^2 I}{N} + (r - eI)\beta$$
$$\dot{J} = rJ - eIJ \quad \dot{I} = \frac{JI}{N} - \gamma I$$
$$(J^*, I^*) = \left(\gamma N, \frac{r}{e}\right)$$

CoSIR





$$\dot{\beta} = \frac{\beta^2 I}{N} + (r - eI)\beta + u\beta$$
$$u = -\eta(t)\frac{dL}{dw}\left(\frac{J}{J^*} - 1\right)$$

η: learning rate
w: LV energy
$L(w)$: CLF

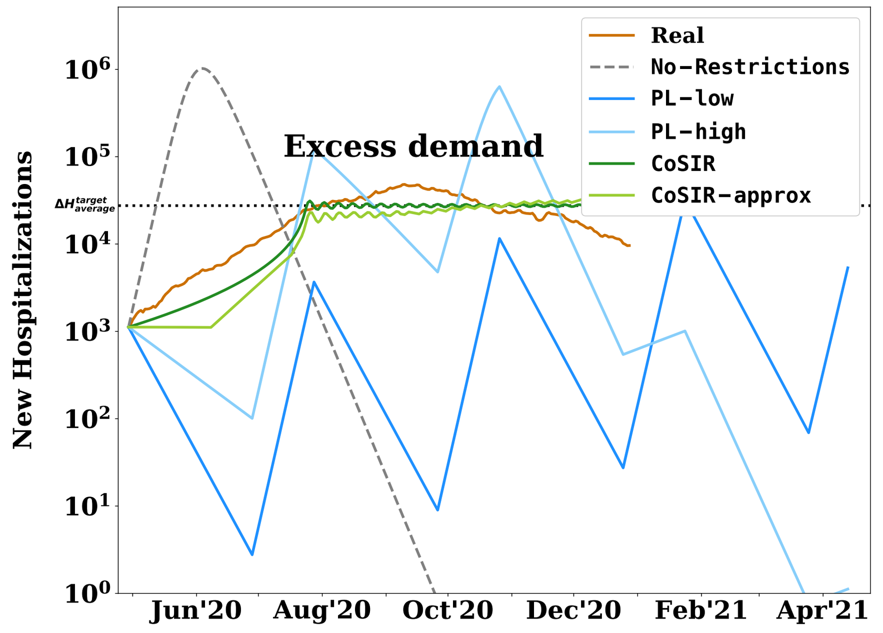
Practical Control Algorithm

1. **Estimate parameters** — Compute γ and β to restriction level mapping from historical case counts
2. **Choose CoSIR parameters** — Choose CoSIR parameters using the system properties
3. **Find optimal β** — Compute the optimal β using CoSIR control policy
4. **Find closest discrete β** — Identify the closest discrete restriction level for optimal β

Experimental Results

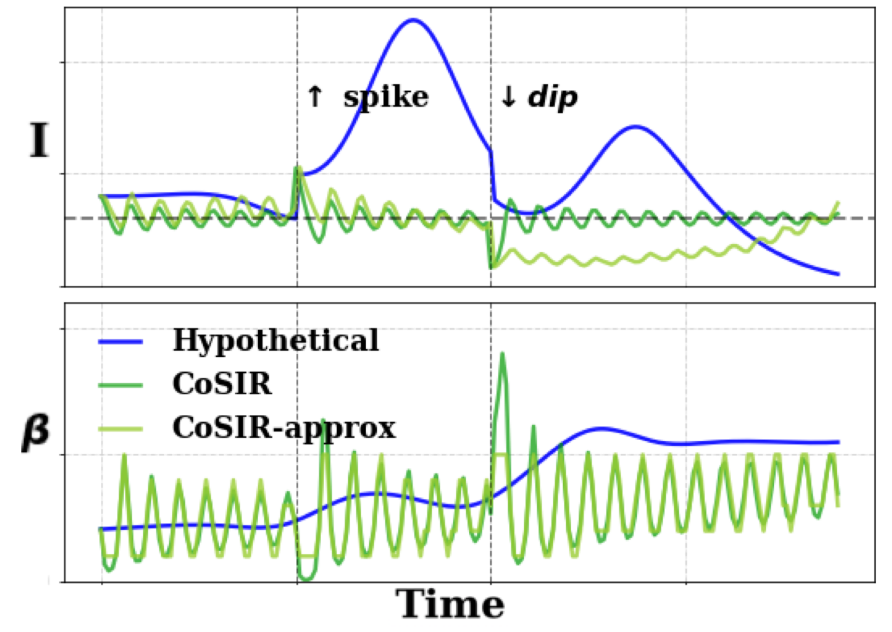
Setup: COVID-19 data (Apr 1-Dec 29, 2020, India), five control policies and real outcome. Parameters were chosen based on manageable hospital inflow.

Control Policies	Description
`No-Restrictions`	Constant $\beta = 0.44$
`PL-high`	60 day lockdown ($\beta = 0.16$) followed by 30 day relaxation ($\beta = 0.44$)
`PL-low`	60 day lockdown ($\beta = 0.1$) followed by 30 day relaxation ($\beta = 0.44$)
`CoSIR`	β follows CoSIR control equation
`CoSIR-approx`	Approximation of CoSIR β with 10 equal levels from 0.1 to 0.55



Hospital Influx

CoSIR variants are closest to the target hospitalization levels. To optimize utilization, we choose β based on available medical capacity and varying susceptibility.



Adaptability

CoSIR variants stabilize infections and adapt to sudden upward ($t = 50$) or downward perturbations ($t = 100$) and continue pushing towards the equilibrium.

Note: Hospitalization influx in SIR is given by $\gamma I \times hospitalization\ ratio$, Real hospitalizations are obtained by appropriate scaling of the reported active cases accounting for under reporting

Future Work

- Design of new epidemic control techniques using CLFs
- Extensions to other compartmental models and control variables
- Extensions to other Hamiltonian dynamical systems